

# **Modélisation des décharges plasmas froids à basse pression**

**(Modeling of low pressure plasma discharges)**

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# Modelling of low-temperature plasmas

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- Particles described separately per species
  - Electrons : energy absorption & ionization
  - Ions : influence electron motion, surface treatment
  - Excited neutrals : stepwise ionization, plasma chemistry
  - Gas neutrals
  
- Classical mechanics
  - Particle approach : Newton's equations + averaging
  - Macroscopic approach : fluid equations
  
- Electromagnetic interaction described via Maxwell equations
  - Electron-ion coupling : Poisson equation
  - Applied field : DC, RF, pulsed, microwave
  
- Collisions treated by input data from experiments & quantum-mechanics  
Cross sections, transport coefficients, rate coefficients  
Mainly with gas  
Many uncertainties / unknowns

# Particle approach

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- Sample individual particles from total population
- Simulate trajectories
- Take statistical averages

- Newton's equations of motion:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

macroscopic fields due  
to collective particles  
(plasma + external)

Self-consistent description  
of plasma fields requires to  
follow a large number of  
particles simultaneously, e.g.  
PIC method

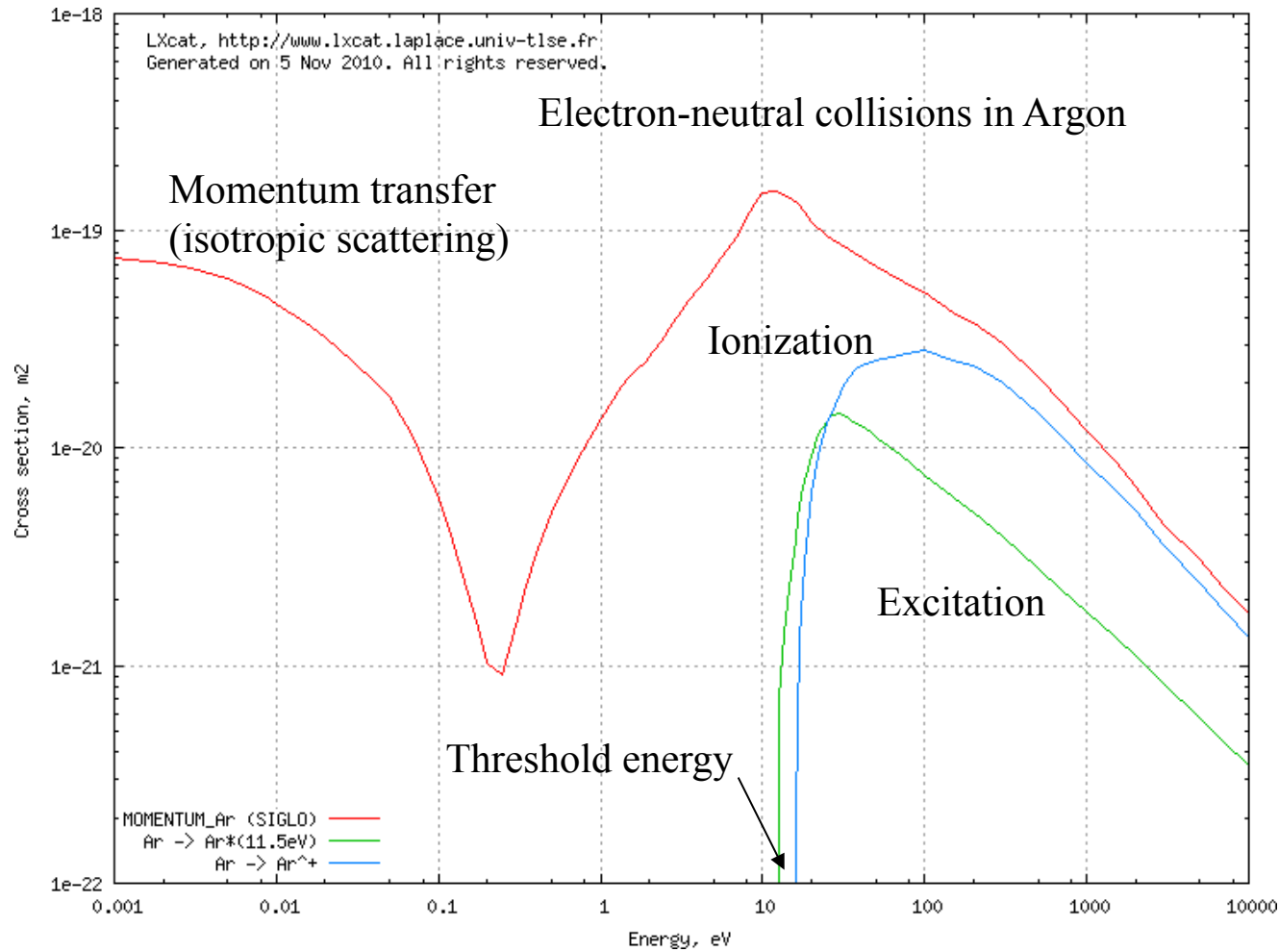
- Collision sampling from probability distributions (Monte Carlo)

collision probability per unit time:  
(= collision frequency)

$$\nu = N\sigma(\mathbf{v}_{\text{rel}})\mathbf{v}_{\text{rel}}$$

target density      cross section      relative velocity

# Cross sections

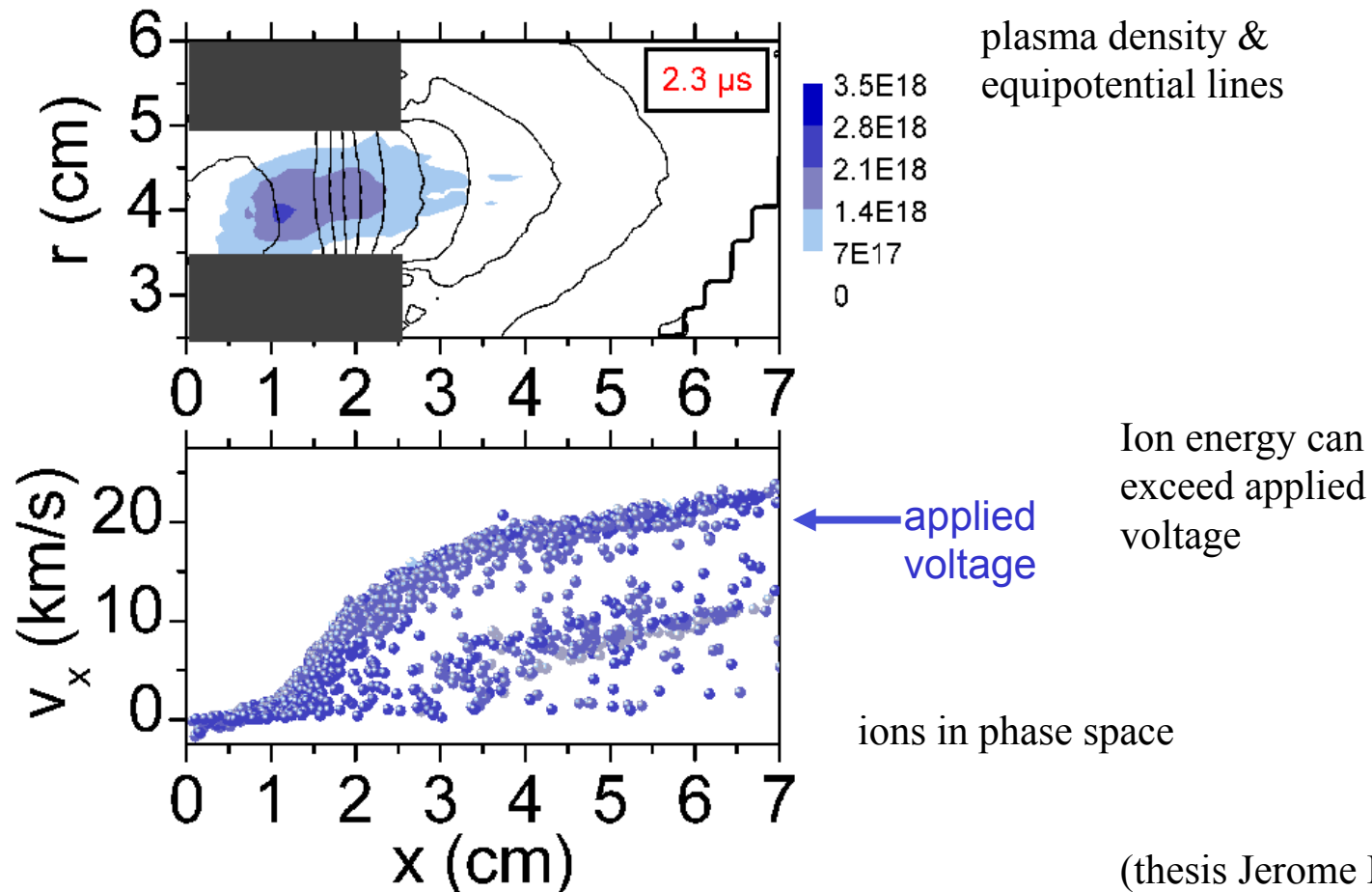


Electron laboratory energy  $\cong$  relative energy

$$\varepsilon = \frac{1}{2} m_e v_e^2 \cong \frac{1}{2} m_{\text{red}} v_{\text{rel}}^2$$

# Hall-effect thruster simulation

Discharge shows instabilities, e.g. transit time oscillations:



(thesis Jerome Barreilles)

# Boltzmann equation

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- Distribution function  $f(t, \mathbf{x}, \mathbf{v})$  = density of particles in phase space
- Spatio-temporal evolution of  $f$  described by Boltzmann equation:

$$\frac{\partial \mathbf{f}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{f} + \frac{\mathbf{q}}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} \mathbf{f} = \mathbf{C}[\mathbf{f}] \quad \longleftarrow \begin{array}{l} \text{collision} \\ \text{operator} \end{array}$$

- Simplify by approximations:
  - Homogeneous approach
  - Nonlocal approach
  - Two-term velocity expansion
  - Velocity-moment approach (fluid equations)
  - ...

# Homogeneous Boltzmann equation

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- Electrons in homogeneous field in state steady:

Spherical harmonics expansion

$$f(\mathbf{x}, \mathbf{v}, t) \cong f_0(\mathbf{v}) + \cos\theta f_1(\mathbf{v}) + \dots$$

$\uparrow$  isotropic component (= EEDF)       $\uparrow$  angle with E       $\swarrow$  anisotropic component

- Two-term homogeneous Boltzmann equation:

$$-\frac{e^2 (E/N)^2}{3m_e^2} \frac{\partial}{\partial \mathbf{v}} \left( \frac{\mathbf{v}}{\sigma_m} \frac{\partial f_0}{\partial \mathbf{v}} \right) = C_0[f_0] \qquad f_1 = \frac{e(E/N)}{m_e \sigma_m} \frac{\partial f_0}{\partial \mathbf{v}}$$

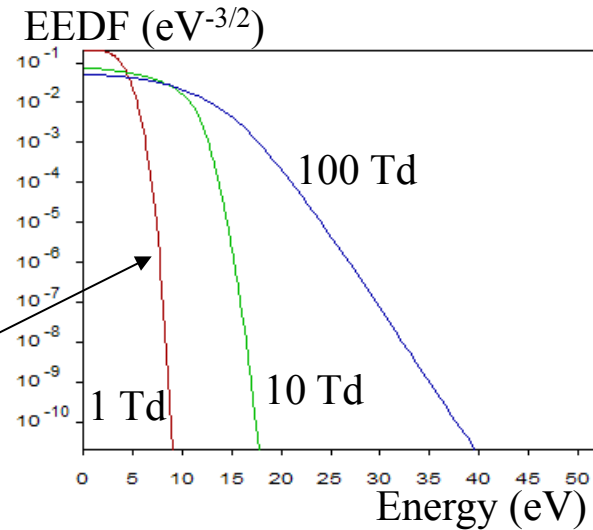
- Yields velocity distribution and all velocity-related quantities (EEDF, mean velocity, mean energy, rate coefficients) as a function of reduced field E/N

# Homogeneous BE results in Argon

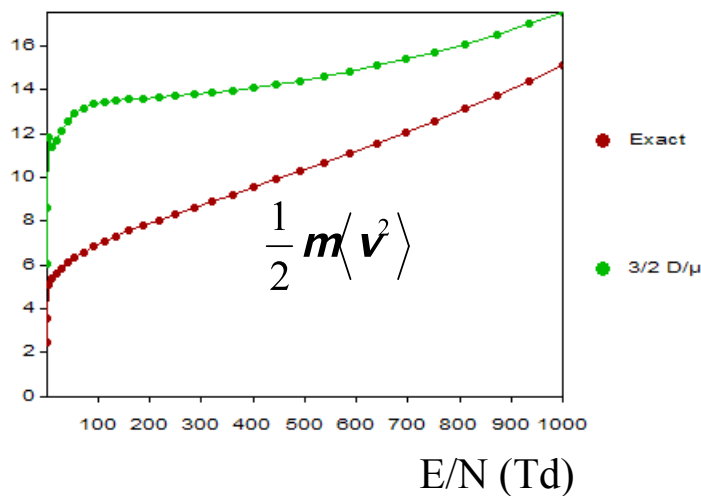
Freeware code BOLSIG+

[www.bolsig.laplace.univ-tlse.fr](http://www.bolsig.laplace.univ-tlse.fr)

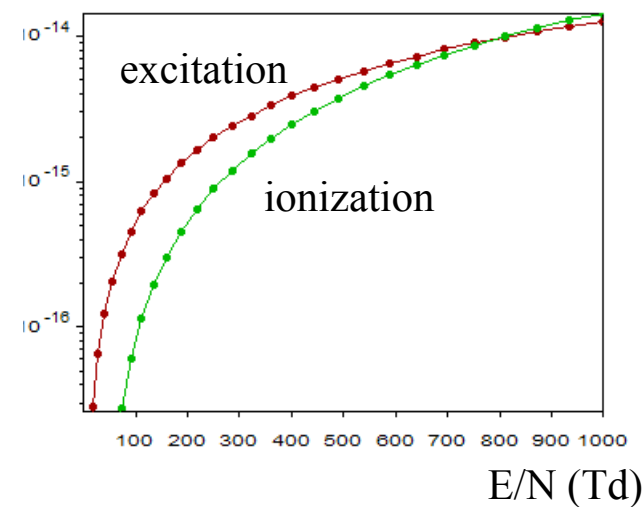
Non-Maxwellian EEDF!



Mean energy (eV)



Rate coefficient ( $\text{m}^3/\text{s}$ )  $k = \langle \sigma(v) v \rangle$





# Fluid approach

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- Macroscopic quantities, averaged over velocity space

Particle density:  $n(\mathbf{x}, t) = \int \int \mathbf{f}(\mathbf{x}, \mathbf{v}, t) d\mathbf{v}$

Mean velocity:  $\mathbf{w} = \langle \mathbf{v} \rangle = \frac{1}{n} \int \int \mathbf{v} f d\mathbf{v}$

Mean energy:  $\varepsilon = \frac{m}{2en} \langle v^2 \rangle = \frac{m}{2en} \int \int v^2 f d\mathbf{v}$

- Macroscopic transport equations = velocity moments of BE

$$\int \int \left( \frac{\partial \mathbf{f}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{f} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} \mathbf{f} \right) \mathbf{v}^m d\mathbf{v} = \int \int \mathbf{A}[\mathbf{f}] \mathbf{v}^m d\mathbf{v}$$

# Fluid equations

- Continuity equation: particle conservation

$$\frac{\partial n}{\partial t} + \nabla \cdot (\underbrace{n\mathbf{w}}_{\text{flux}}) = \underbrace{S}_{\text{source}} \quad S = \sum_I N_i n_i n_{2i} k_i \quad k_i = \langle \sigma_i(\mathbf{v}) \mathbf{v} \rangle$$

rate coefficient

- Momentum equation:

$$m \frac{\partial n\mathbf{w}}{\partial t} + m \nabla \cdot (n\mathbf{w} \otimes \mathbf{w}) + \nabla \cdot \mathbf{P} - qn(\mathbf{E} + \mathbf{w} \times \mathbf{B}) = - \underbrace{\bar{m}_m}_{\substack{\text{momentum transfer} \\ \text{frequency}}} n\mathbf{w} \quad \text{collisions}$$

inertia                      pressure tensor:  $\mathbf{P} = m \int (\mathbf{v} - \mathbf{w}) \otimes (\mathbf{v} - \mathbf{w}) f d\mathbf{v}$

- Short mean free path: drift-diffusion approximation:

$$\text{flux } n\mathbf{w} = \frac{q}{m\bar{v}_m} n\mathbf{E} - \frac{e}{m\bar{v}_m} \nabla(nT) \equiv \frac{q}{|q|} \underbrace{\mu}_{\text{mobility}} n\mathbf{E} - \nabla(Dn)$$

mobility                      diffusion coefficient

# Minimal self-consistent model (high pressure)

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- Electron & ion continuity:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{w}_e) = KNn_e \qquad \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{w}_i) = KNn_e$$

- Electron & ion drift-diffusion:

$$n_e \mathbf{w}_e \equiv -\mu_e n_e \mathbf{E} - D_e \nabla n_e \qquad n_i \mathbf{w}_i \equiv \mu_i n_i \mathbf{E} - D_i \nabla n_i$$

- Poisson (ambipolar + applied field):

$$\varepsilon_0 \nabla \cdot \mathbf{E} = -\varepsilon_0 \nabla^2 \Phi = e n_i - e n_e \qquad \mathbf{E} = -\nabla \Phi$$

- Local field approximation: transport coefficients and ionization rate are functions of reduced field E/N (from experiments are homogeneous BE)
- Boundary conditions for the particle fluxes

# Boundary conditions

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- Particle flux toward the wall

$$\text{net flux} \longrightarrow \mathbf{n}w_{\perp} = \mathbf{n}w_w - \Gamma_w \longleftarrow \text{particles coming from wall due to reflection and creation (emission)}$$

$\swarrow$  particles moving to the wall

- Effective velocity of incident particles obtained from kinetic considerations

$$w_w = \frac{1}{4} \sqrt{\frac{8eT}{\pi m}} + \max(\text{sgn}(\mathbf{q}\mu\mathbf{E}_{\perp}), 0)$$

$\uparrow$   
 $\frac{1}{4}$  or  $\frac{1}{2}$  or ?

- Flux coming from the wall obtained from incident fluxes

$$\Gamma_w = \sum_s \gamma_s \mathbf{n}_s w_{w,s}$$

$\nwarrow$  emission coefficient  
 sum over all species

- Equating to drift-diffusion  $\rightarrow$  mixed boundary condition, e.g.

$$(\text{sgn}(\mathbf{q}\mu\mathbf{E}_{\perp} - w_w) \mathbf{n} - D \nabla_{\perp} \mathbf{n} + \Gamma_w = 0$$

# Numerics

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- Integrate equations sequentially in time  $t^{k+1} = t^k + \Delta t$

- Implicit update of drift-diffusion flux:

$$\frac{n^{k+1} - n^k}{\Delta t} + \nabla \cdot (\pm \mu E n^{k+1} - D \nabla n^{k+1}) = S$$

k = time index

No CFL time step constraints  $\Delta t < \left( W/\Delta x + 2D/\Delta x^2 \right)^{-1}$

- Exponential scheme for drift-diffusion flux:

i = space index

$$\left( Wn - D \frac{\partial n}{\partial x} \right)_{i+1/2} = \frac{W_{i+1/2}}{1 - \exp(-z)} n_i + \frac{W_{i+1/2}}{1 - \exp(z)} n_{i+1} \quad z = \frac{W_{i+1/2} \Delta x}{D_{i+1/2}} = \frac{\pm \mu E_{i+1/2} \Delta x}{D_{i+1/2}}$$

For electrons in ambipolar sheath:  
drift  $\approx$  diffusion  $\rightarrow$  Boltzmann relation:

$$\frac{n_{i+1}}{n_i} = \exp(z) \cong \exp\left(\frac{\Phi_{i+1} - \Phi_i}{T}\right)$$

# Semi-implicit Poisson method

- Charged particle transport strongly coupled with Poisson equation:

$$\nabla^2 \Phi^{k+1} = \frac{\mathbf{e}}{\epsilon_0} (\mathbf{n}_e - \mathbf{n}_i)$$

$$\frac{\mathbf{n}^{k+1} - \mathbf{n}^k}{\Delta t} + \nabla \cdot (\mp \mu \mathbf{n}^{k+1} \nabla \Phi^{k+1} - \mathbf{D} \nabla \mathbf{n}^{k+1}) = \mathcal{S}$$

Coupling time constant  
(Maxwell relaxation time)

$$\tau_d = \frac{\epsilon_0}{\mathbf{e}(\mu_e \mathbf{n}_e + \mu_i \mathbf{n}_i)} \cong \frac{\epsilon_0}{\mathbf{e} \mu_e \mathbf{n}_e}$$

- Avoid time step constraint  $\Delta t < \tau_d$  by space charge prediction:

$$\nabla^2 \Phi^{k+1} = \frac{\mathbf{e}}{\epsilon_0} (\tilde{\mathbf{n}}_e^{k+1} - \tilde{\mathbf{n}}_i^{k+1}) \quad \tilde{\mathbf{n}}^{k+1} = \mathbf{n}^k + \Delta t (\mathcal{S} - \nabla \cdot (\mp \mu \mathbf{n}^k \nabla \Phi^{k+1} - \mathbf{D} \nabla \mathbf{n}^k))$$

Modified Poisson equation:

$$\nabla \cdot ((1 + \chi) \nabla \Phi^{k+1}) = \nabla \cdot (\chi \nabla \Phi^k) + \frac{\mathbf{e}}{\epsilon_0} (2\mathbf{n}_e^k - \mathbf{n}_e^{k-1} - 2\mathbf{n}_i^k + \mathbf{n}_i^{k-1})$$

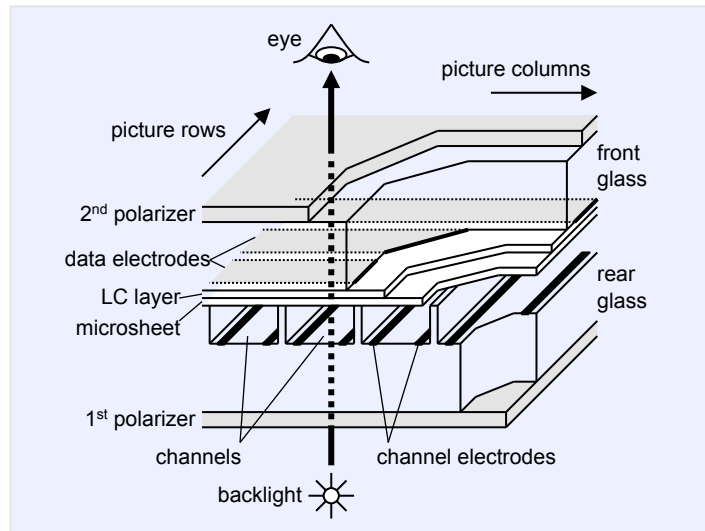
Extrapolated space charge density

Semi-implicit susceptibility:  $\chi = \frac{\mathbf{e} \Delta t}{\epsilon_0} (\mu_e \mathbf{n}_e + \mu_i \mathbf{n}_i) = \frac{\Delta t}{\tau_d}$

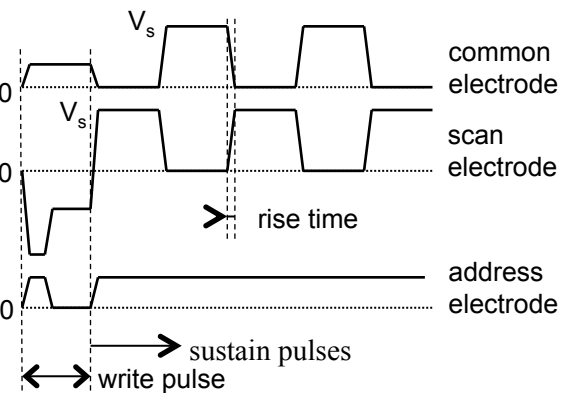
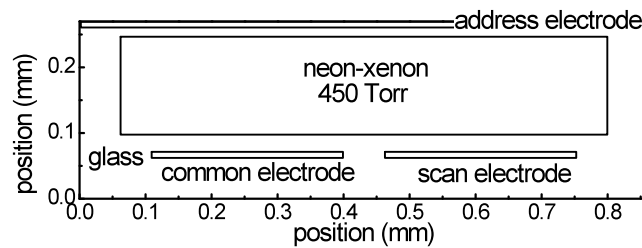
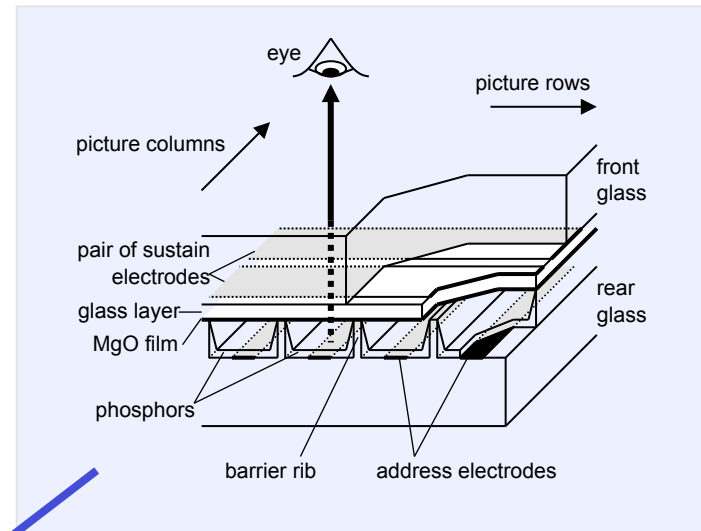
Semi-implicit terms cancel in steady state!

# Microdischarges for display technology

Plasma Addressed Liquid Crystal  
(Philips Eindhoven 1995-1998)



Plasma Display Panel (PDP)  
(Philips Aachen 1998-2001)

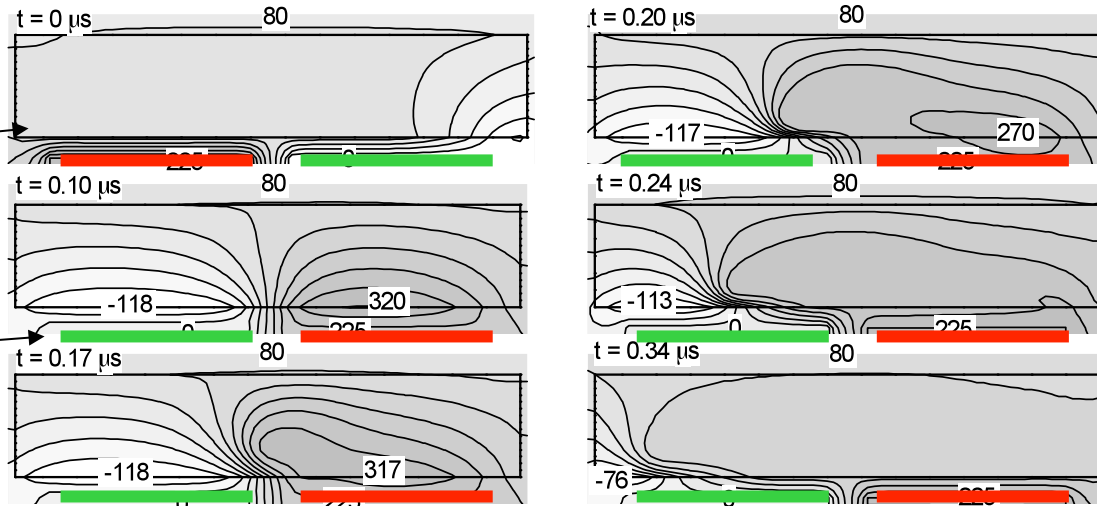


# Coplanar PDP simulation

## Electric potential

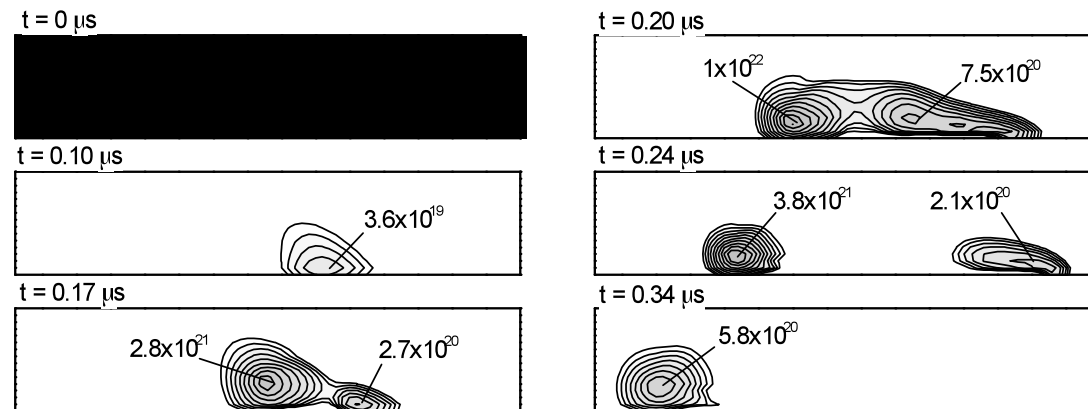
Field screened by surface charges from previous discharge

Polarity change creates new discharge



Discharge stopped by surface charges

## UV emission rate

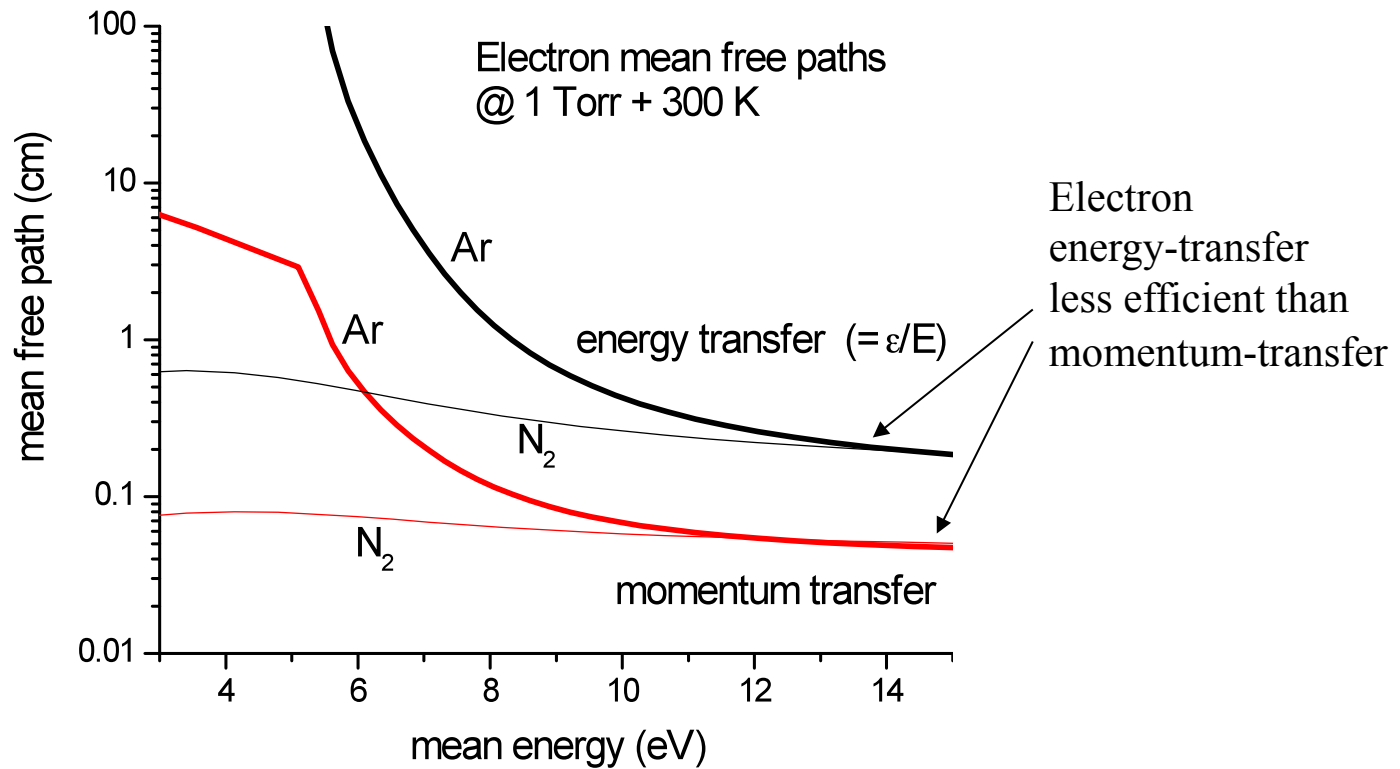




# Low pressure

Compare mean free path with macroscopic length scales (plasma size etc)

Mfp inversely proportional to pressure



# Electron energy equation

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- Long energy-transfer mean-free-path: local field approximation not valid

~~$$E/N \rightarrow f_0, \bar{\varepsilon}, \mu_e, D_e, K$$~~

- Solve mean energy from energy equation:

$$\frac{\partial n_e \bar{\varepsilon}}{\partial t} + \frac{5}{3} \nabla \cdot (n_e w_e \bar{\varepsilon} - n_e D_e \nabla \bar{\varepsilon}) = -en_e w_e \cdot \mathbf{E} + P_{\text{ext}} - n_e N \kappa$$

↑ thermal conduction
 ↑ work of electrostatic field
 ↑ inductive / MW power
 ← collisional losses

- Parametrise electron transport coefficients & rates as a function of electron mean energy

$$\bar{\varepsilon} \rightarrow \mu_e, D_e, K, \kappa$$

- Maxwellian EEDF:

$$\bar{\varepsilon} = \frac{3}{2} T_e$$

$$D_e = \mu_e T_e$$

Einstein relation

# Momentum equation

- Long momentum-transfer mean-free-path: drift-diffusion not valid, reconsider momentum equation:

$$m \frac{\partial \mathbf{nw}}{\partial t} + m \bar{\mathbf{v}} \cdot (\mathbf{nw} \otimes \mathbf{w}) + \nabla \cdot \mathbf{P} - qn(\mathbf{E} + \mathbf{w} \times \mathbf{B}) = - m \bar{\mathbf{v}}_m \mathbf{nw}$$

- Electrons: isotropic due to ambipolar trapping  $\rightarrow$  neglect  $\mathbf{w}$  terms

Boltzmann equilibrium:  $T_e \nabla n_e \approx -en_e \mathbf{E} \longrightarrow n_e = n_0 \exp\left(\frac{\Phi - \Phi_0}{T_e}\right)$   
 Drift-diffusion equilibrium:  $D_e \nabla n_e \approx -\mu_e n_e \mathbf{E} \longrightarrow$  Boltzmann relation

- Ions: very anisotropic  $\rightarrow$  neglect pressure (& substitute continuity equation)

$$\frac{\partial \mathbf{w}_i}{\partial t} + (\mathbf{w}_i \cdot \nabla) \mathbf{w}_i + \left( \bar{v}_i + \frac{\mathbf{S}_i}{n_i} \right) \mathbf{w}_i = \frac{e}{m_i} \mathbf{E}$$

$\uparrow$  inertia: memory in time & space  
 $\uparrow$  ion creation at zero velocity

If  $\mathbf{w} \parallel \mathbf{E}$ :

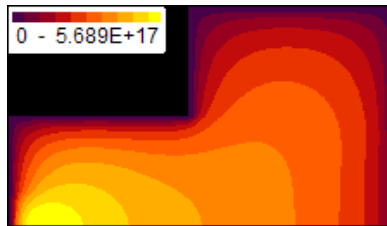
$$\nabla \left( \frac{1}{2} m_i w_i^2 + e\Phi \right) + \left( \bar{v}_i + \frac{\mathbf{S}_i}{n_i} \right) m_i w_i = 0$$

# Low pressure ambipolar plasma transport

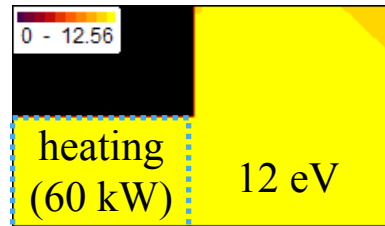
Self-consistent description of sheath & presheath: Poisson equation

Example without magnetic field:

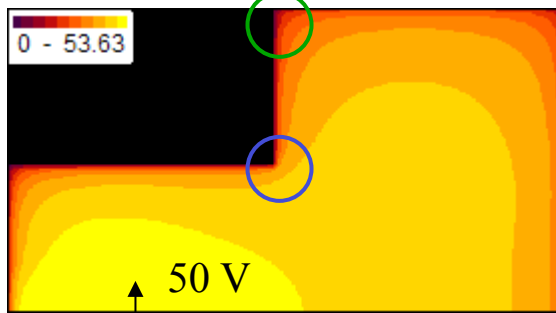
plasma density



e temperature (eV)

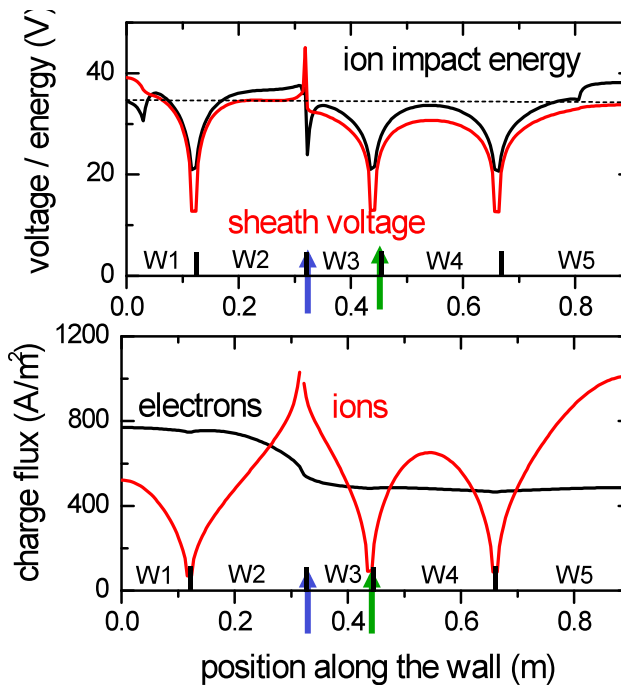


potential (V)



plasma potential

sheath voltage  
& ion energy  
vary along the  
walls!



$$\Phi_p \approx T_e \left( 1 + \frac{1}{2} \ln \left( \frac{m_i}{2\pi m_e} \right) \right) \approx 4 T_e$$

# Dense plasmas: quasineutral approach

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- Compare sheath size (Debye length) with plasma size  $\lambda_D = \sqrt{\frac{\epsilon_0 T_e}{en_e}}$
- Thin sheath  $\rightarrow$  eliminate Poisson's equation using quasineutrality:

Solve electric field from  
electron conservation /  
current conservation:

$$\nabla \cdot (-\mu_e \mathbf{n}_e \mathbf{E} - \mathbf{D}_e \nabla n_e) = \mathcal{S}_e - \frac{\partial n_e}{\partial t} = \nabla \cdot (\mathbf{n}_i \mathbf{w}_i)$$

$n_e = n_i$

- Drift-diffusion ions: ambipolar diffusion:

Separate ambipolar / external field:

$$\mathbf{E} = \mathbf{E}_{\text{amb}} + \mathbf{E}_{\text{ext}}$$

$$(\mu_e + \mu_i) \mathbf{n}_e \mathbf{E}_{\text{amb}} = -(\mathbf{D}_e - \mathbf{D}_i) \nabla n_e$$

ambipolar diffusion  
coefficient

$$\frac{\partial n_e}{\partial t} - \nabla \cdot \left( \frac{\mu_i \mathbf{D}_e + \mu_e \mathbf{D}_i}{\mu_e + \mu_i} \nabla n_e \right) = \mathcal{S}_e$$

$$\nabla \cdot ((\mu_e + \mu_i) \mathbf{n}_e \mathbf{E}_{\text{ext}}) = 0$$

- Complications at low pressure due to inertia & boundary conditions  
But: semi-implicit Poisson method also works!

# **Hybrid models of magnetized discharge plasmas: fluid electrons + particle ions**

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# Introduction

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Magnetic fields used in low-pressure discharges:

- magnetron
- electron-cyclotron resonance (ECR)
- helicon
- Hall-effect thruster
- etc... (magnetized discharges)

Magnetic field → complex physics

Insight from simple models

# Plan

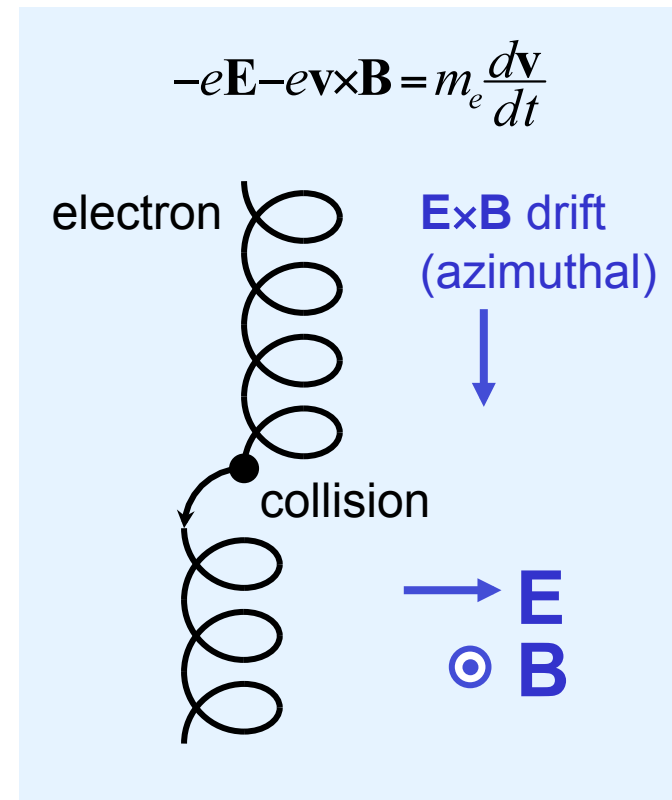
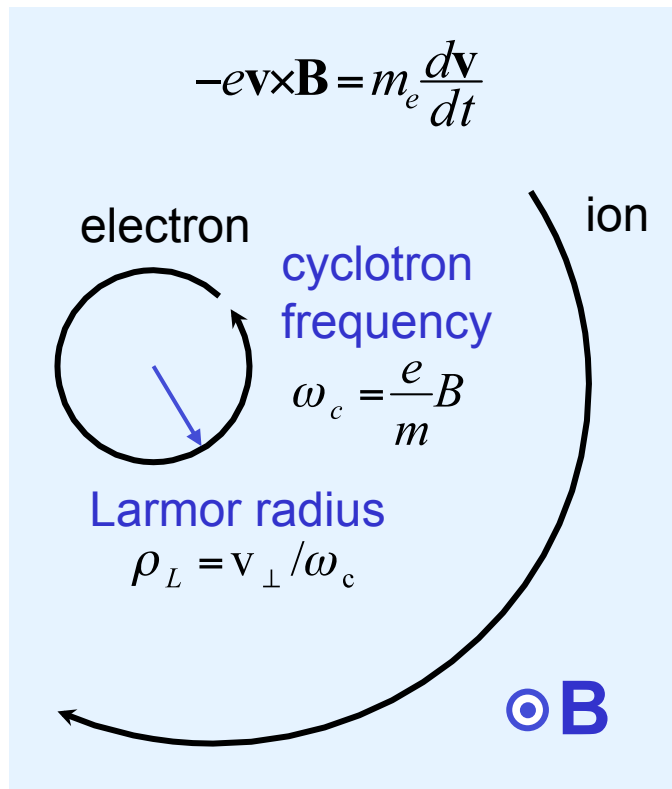
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- Elementary physics
- Modelling
- Limits of modelling
- Illustrative model results:
  - ECR reactor
  - Hall thruster
  - Galathea trap



# Elementary effects of the magnetic field

- Cyclotron motion  $\rightarrow$  confinement
- Perpendicular electric field  $\rightarrow \mathbf{E} \times \mathbf{B}$  drift
- Collisions destroy magnetic confinement



# Typical conditions

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<b>plasma</b>	pressure	0.1 – 10 mTorr
	plasma density	$10^{15} - 10^{19} \text{ m}^{-3}$
	magnetic field	0.001 – 0.1 T
	electron temperature	2 – 20 eV
<b>lengths</b>	Debye length	$10^{-5} - 10^{-3} \text{ m}$
	electron Larmor radius	$10^{-4} - 0.01 \text{ m}$
	ion Larmor radius	0.02 – 5 m
	mean free path	0.01 – 1 m
	plasma size	0.02 – 1 m
<b>frequencies</b>	electron cyclotron	$3 \times 10^8 - 2 \times 10^{10} \text{ s}^{-1}$
	electron collision	$3 \times 10^5 - 10^8 \text{ s}^{-1}$

Long mean free path

Electrons are magnetized → collisions + ionization

Ions have only few collisions

Magnetic field not influenced by plasma

# Modelling

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Low pressure → particle-in-cell (PIC):

- electron and ion trajectories
- space charge electric fields

K. A. Ashtiani *et al*, J. Appl. Phys. 78 (4), 2270-2278 (1995).

S. Kondo and K. Nanbu, J. Phys. D: Appl. Phys. **32**, 1142-1152 (1999).

J. C. Adam *et al*, Phys. Plasmas **11** (1), 295-305 (2004).

Magnetized PIC models cumbersome:

- high plasma density → small time steps, small cells
- important 2D effects

→ interest in simpler faster models

→ describe electrons by collisional fluid equations

# Electron fluid equations

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- Electron conservation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma_e = S$$

density      flux

ionisation  
source

- Anisotropic flux

$$\Gamma_e = \mu n_e \nabla \Phi - \mu \nabla (n_e T_e)$$

drift                  diffusion

- Mobility tensor  
(classical theory)

$$\mu_{\perp} = \frac{\nu^2}{\nu^2 + \omega_c^2} \mu_{\parallel} = \frac{e\nu/m_e}{\nu^2 + \omega_c^2}$$

collision  
frequency

cyclotron frequency

perpendicular mobility  $\ll$  parallel mobility

# Magnetized drift-diffusion equation

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Hall parameter  
↓

$$\mathbf{n}_e \mathbf{w}_e - \mu \mathbf{B} \times (\mathbf{n}_e \mathbf{w}_e) = \overset{\text{drift}}{\mu \mathbf{n}_e \nabla \Phi} - \overset{\text{diffusion}}{\mu \nabla (\mathbf{n}_e T_e)} \equiv \mathbf{G}$$

Take cross product and dot product with  $\mathbf{B}$  and combine

$$\mathbf{n}_e \mathbf{w}_e = \frac{1}{1 + (\mu \mathbf{B})^2} \left( \mathbf{G} - \mu \mathbf{B} \times \mathbf{G} + \mu^2 (\mathbf{B} \cdot \mathbf{G}) \mathbf{B} \right) \equiv \underbrace{(\mu / \mu)}_{\text{mobility tensor}} \cdot \mathbf{G}$$

Mobility tensor components:

$$\mu_{//} = \mu$$

Parallel transport unaffected

$$\mu_{\perp} = \frac{1}{1 + (\mu \mathbf{B})^2} \mu$$

Perpendicular confinement

$$\mu_{\times} = \pm \frac{\mu \mathbf{B}}{1 + (\mu \mathbf{B})^2} \mu$$

Magnetic drifts

# Hybrid models

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Non-quasineutral scheme:

- ion particles  $\rightarrow n_i$
- electron fluid  $\rightarrow n_e$
- Poisson  $\rightarrow \Phi$

$$\varepsilon_0 \nabla^2 \Phi = e(n_e - n_i)$$

no plasma oscillations  
 $\rightarrow$  large time steps

Quasineutral scheme:

- ion particles  $\rightarrow n_i = n_e$
- electron fluid  $\rightarrow \Phi$

no sheaths  $\rightarrow$  large cells

$$\nabla \cdot (\mu n_e \nabla \Phi - \mu \nabla (n_e T_e)) = \nabla \cdot \Gamma_i \quad (\text{Ohm's law})$$

R. K. Porteous *et al*, Plasma Sources Sci. Technol. **3**, 25-39 (1994).

J. M. Fife, Ph. D. thesis, MIT, 1998.

G. J. M. Hagelaar *et al*, J. Appl. Phys. **91** (9), 5592-5598 (2002).

# Limits of the electron equations

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- Anomalous transport  $\perp \mathbf{B} \rightarrow$  empirical parameters

$$ev/m_e (v^2 + \omega_c^2) < \mu_{\perp} < 1/16B$$

classical mobility      ?      Bohm mobility

- Non-local effects  $// \mathbf{B}$ : inertia, mirror confinement  
But: flux  $// \mathbf{B}$  limited by boundaries

$$\mu_{\parallel} n_e \nabla_{\parallel} \Phi \approx \mu_{\parallel} \nabla_{\parallel} (n_e T_e)$$

drift

diffusion

$$\rightarrow \Phi(\mathbf{r}) = \Phi^*(\lambda) + T_e(\lambda) \ln(n_e(\mathbf{r})/n_0) \quad (\text{Boltzmann})$$

potential = constant + diffusion term

Magnetic field lines approximately equipotential

# Numerical issues (1)

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Extreme anisotropy  $\rightarrow$  numerical errors tend to destroy the magnetic confinement

electron flux

$$\Gamma_{e,x} = \frac{1+\Omega_x^2}{1+\Omega^2} \left[ \mu_0 n_e \frac{\partial \Phi}{\partial x} - \mu_0 \frac{\partial n_e T_e}{\partial x} \right] + \frac{\Omega_x \Omega_y}{1+\Omega^2} \left[ \mu_0 n_e \frac{\partial \Phi}{\partial y} - \mu_0 \frac{\partial n_e T_e}{\partial y} \right]$$

Hall vector

$$\Omega = \frac{e\mathbf{B}}{m_e \nu}$$

$$|\Omega| = \omega_c / \nu \gg 1$$

drift

diffusion

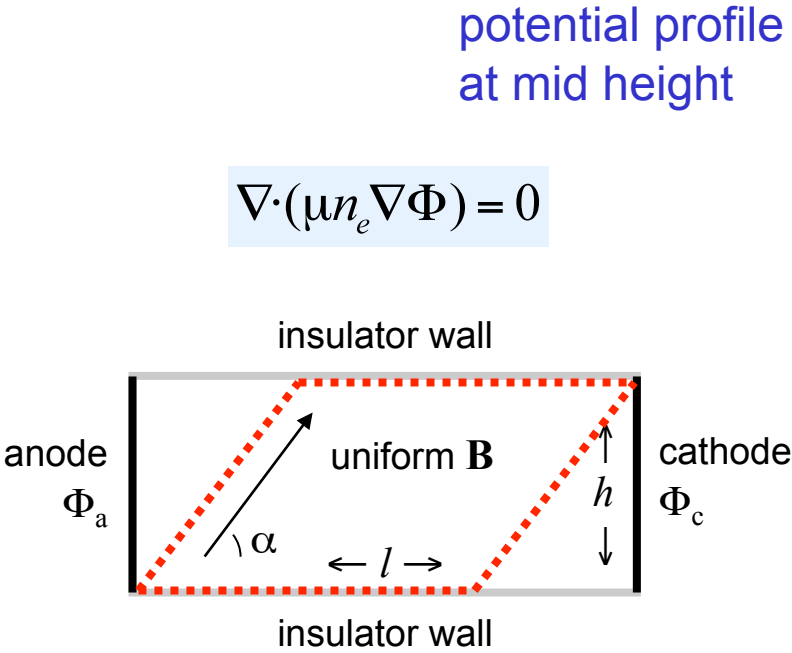
drift

diffusion

longitudinal term and transverse term  
very large and opposite in sign

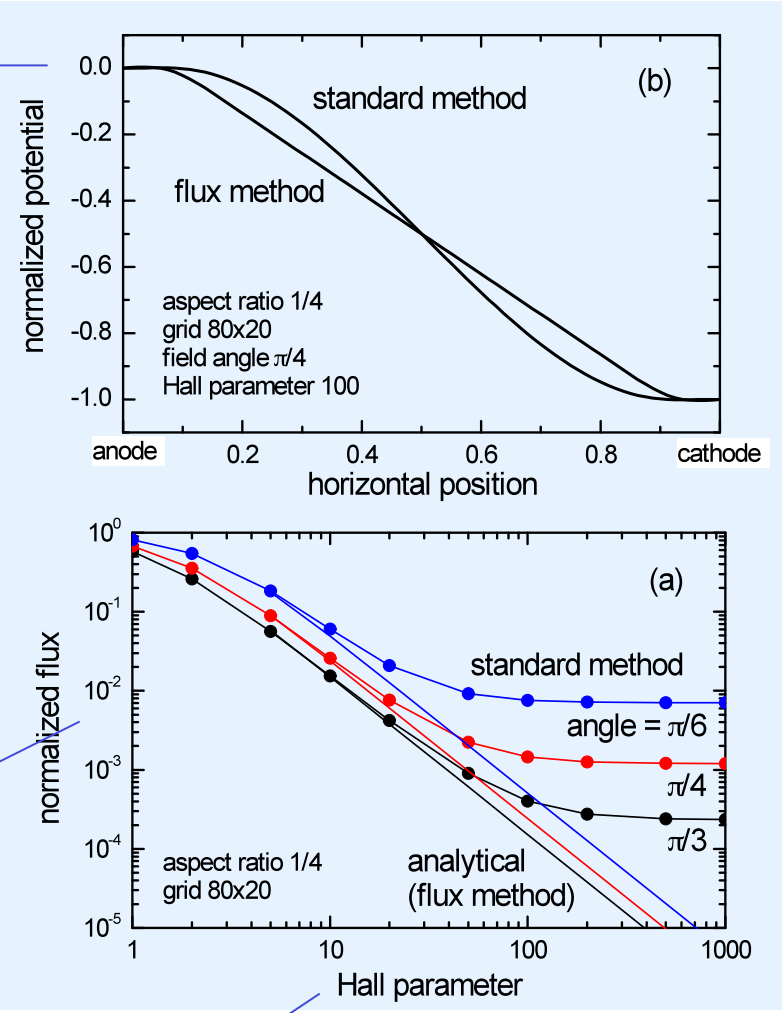


# Numerical issues (2)



potential profile at mid height

electron flux in the middle of the channel



$|\Omega| = [\text{cyclotron frequency}] / [\text{collision frequency}]$

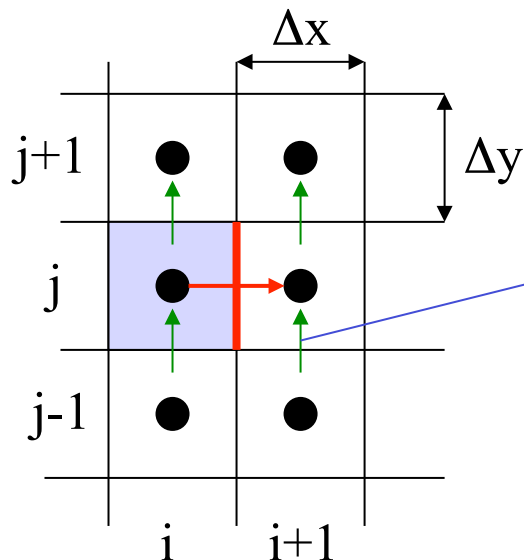
## Numerical issues (3)

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Iterative flux scheme:

interpolate transverse flux rather than transverse field

$$\Gamma_{e,x}^{k+1} = \frac{1}{1 + \Omega_y^2} \left[ \mu_0 n_e \frac{\partial \Phi}{\partial x} - \mu_0 \frac{\partial n_e T_e}{\partial x} \right]^{k+1} + \frac{\Omega_x \Omega_y}{1 + \Omega_y^2} \bar{\Gamma}_{e,y}^k$$



average of 4 surrounding  
transverse fluxes

## Numerical issues (4)

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Coupling with Poisson's equation:  
severe time step constraint for explicit scheme

$$\Delta t < \frac{\epsilon_0}{en_e \mu_{||}} = \frac{v}{\omega_{pe}^2} < 10^{-11} \text{ s}$$

(vs. ion CFL-time  $10^{-8} - 10^{-6} \text{ s}$ )

Semi-implicit scheme:  
Poisson's equation includes prediction of space charge

$$\nabla \cdot ((\epsilon_0 + e\Delta t \mu_{||} n_e) \nabla \Phi - e\Delta t \mu_{||} \nabla (n_e T_e)) = e(n_e - n_i)$$

implicit  
space charge  
prediction

# Examples of model results

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Non-quasineutral hybrid model → sheaths resolved

Fixed:

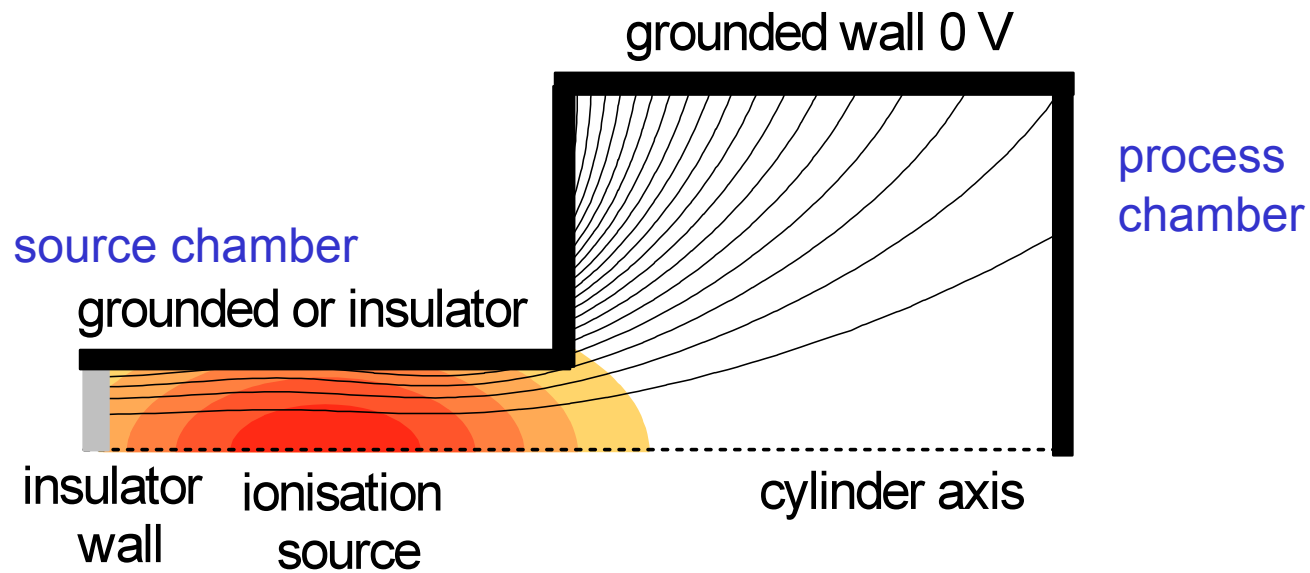
- Gaussian ionisation source
- uniform electron temperature (diffusion)
- electron collision frequency

Calculated:

- electron/ion densities
- electron/ion fluxes, currents
- self-consistent potential

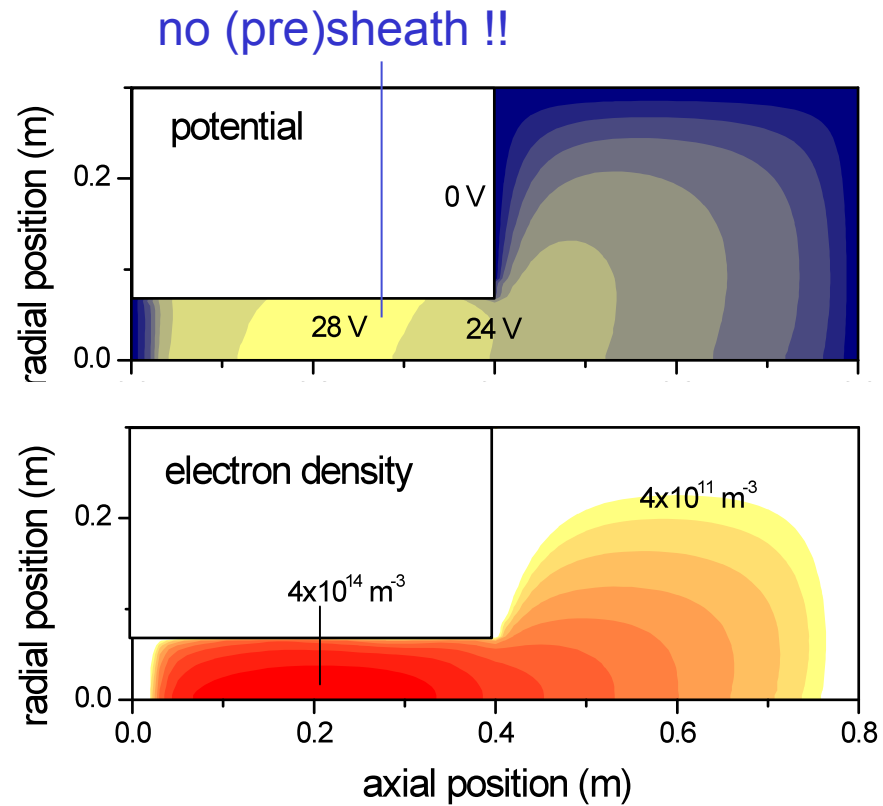
# Example I : Diffusion in ECR reactor

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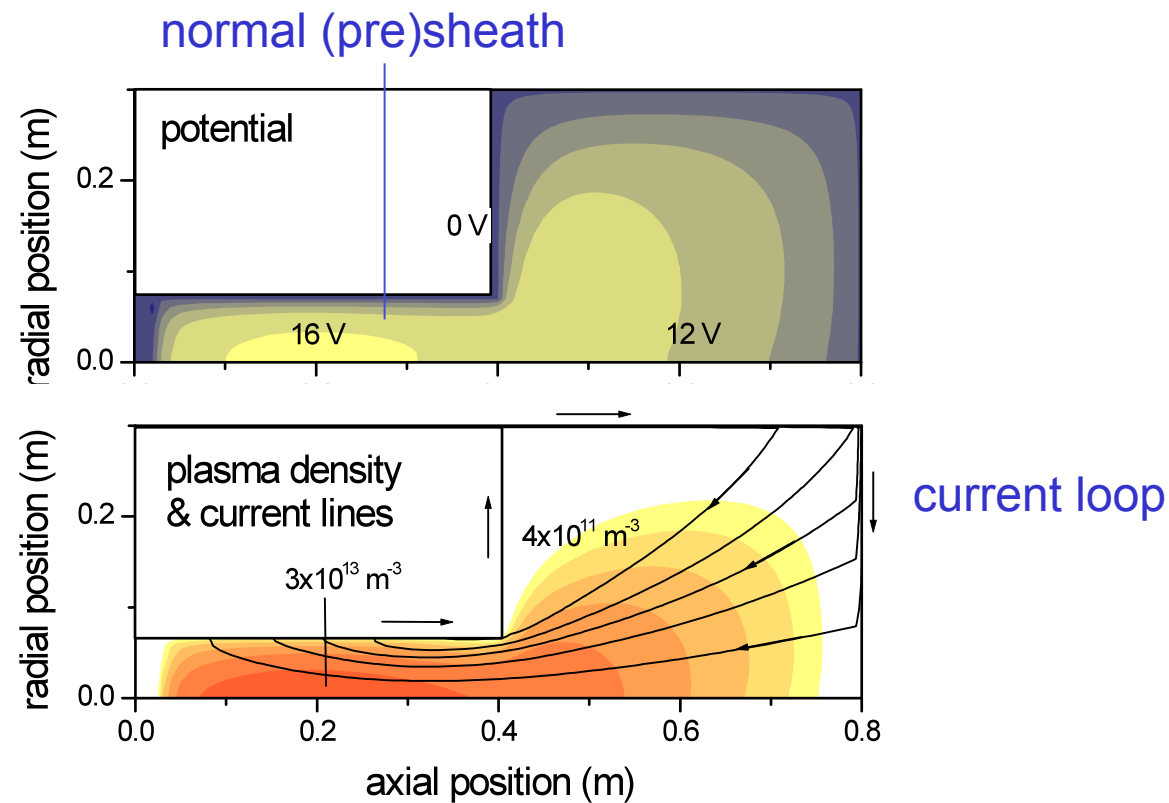
# ECR reactor with dielectric wall

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Magnetic confinement reduces loss to source wall

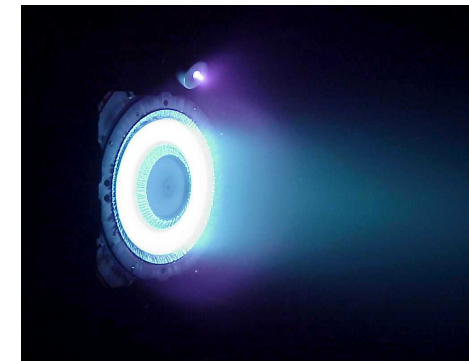
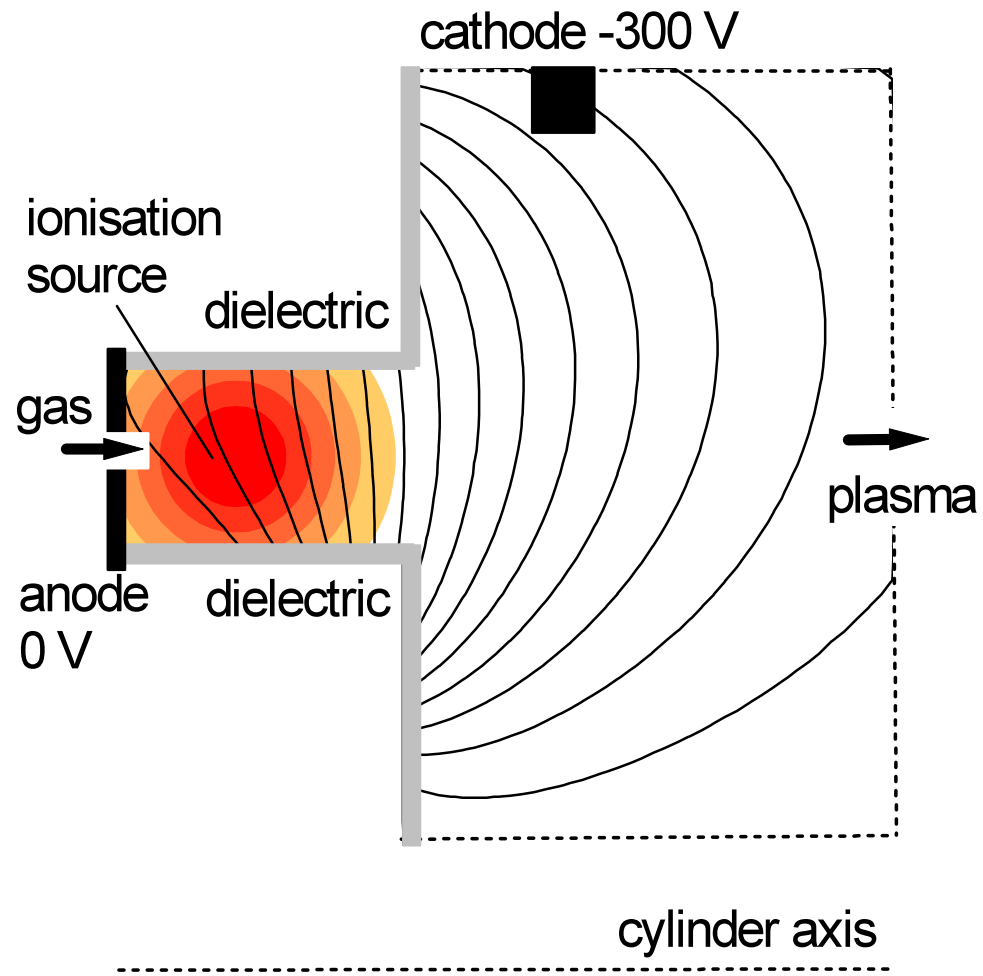
# ECR reactor with grounded wall



Magnetic confinement shortcircuited by walls

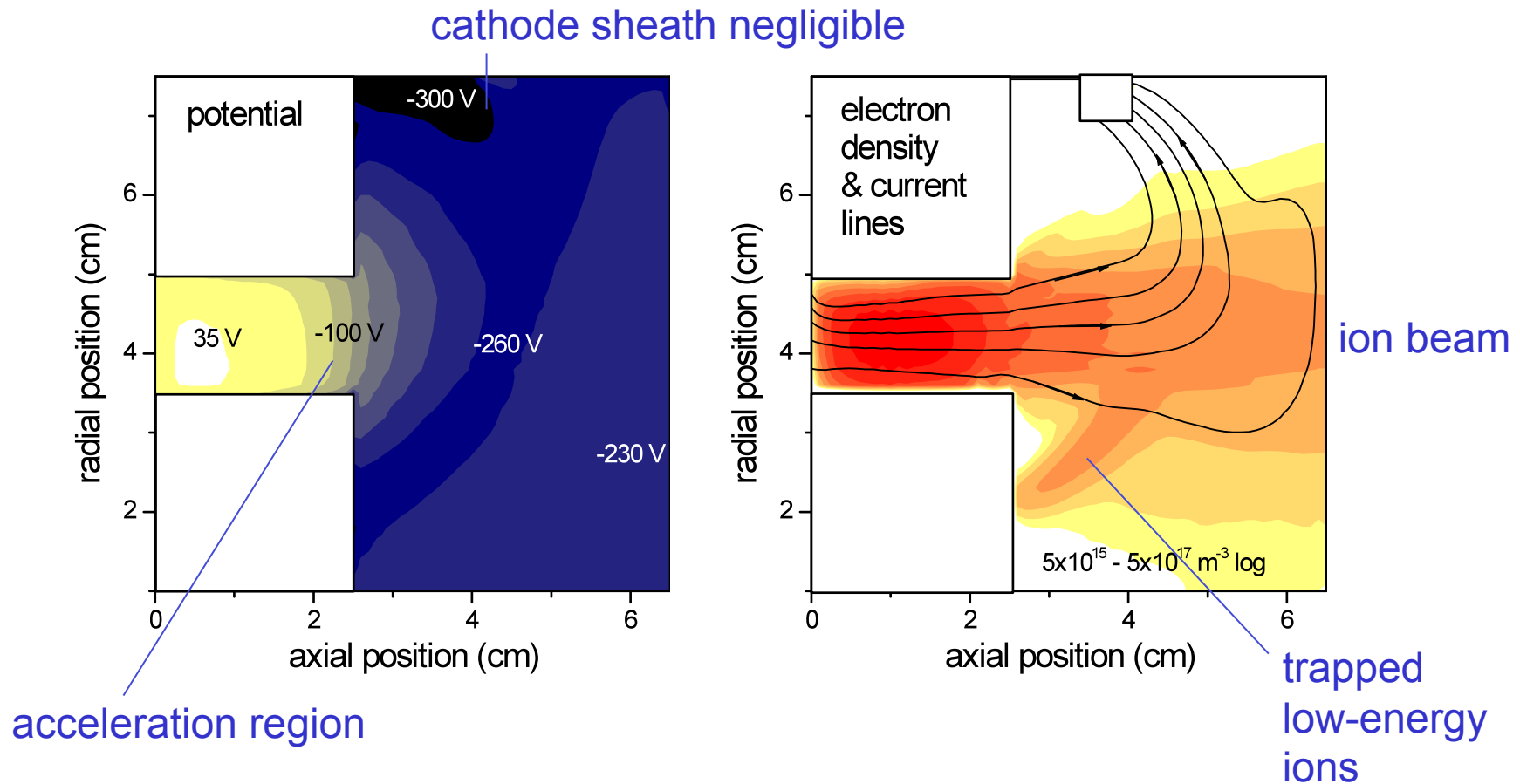
A. Simon, Phys. Rev. **98** (2), 317-318 (1955).

# Example II : Hall-effect thruster



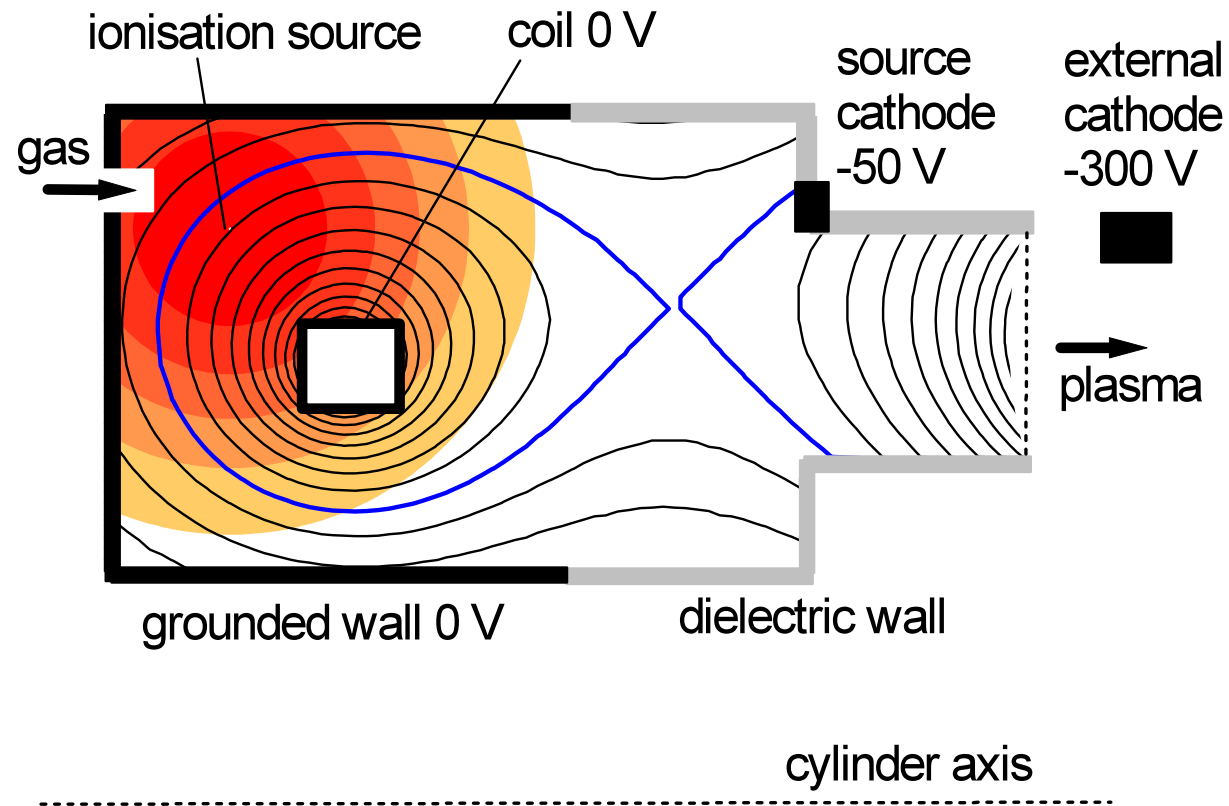


# Hall-effect thruster



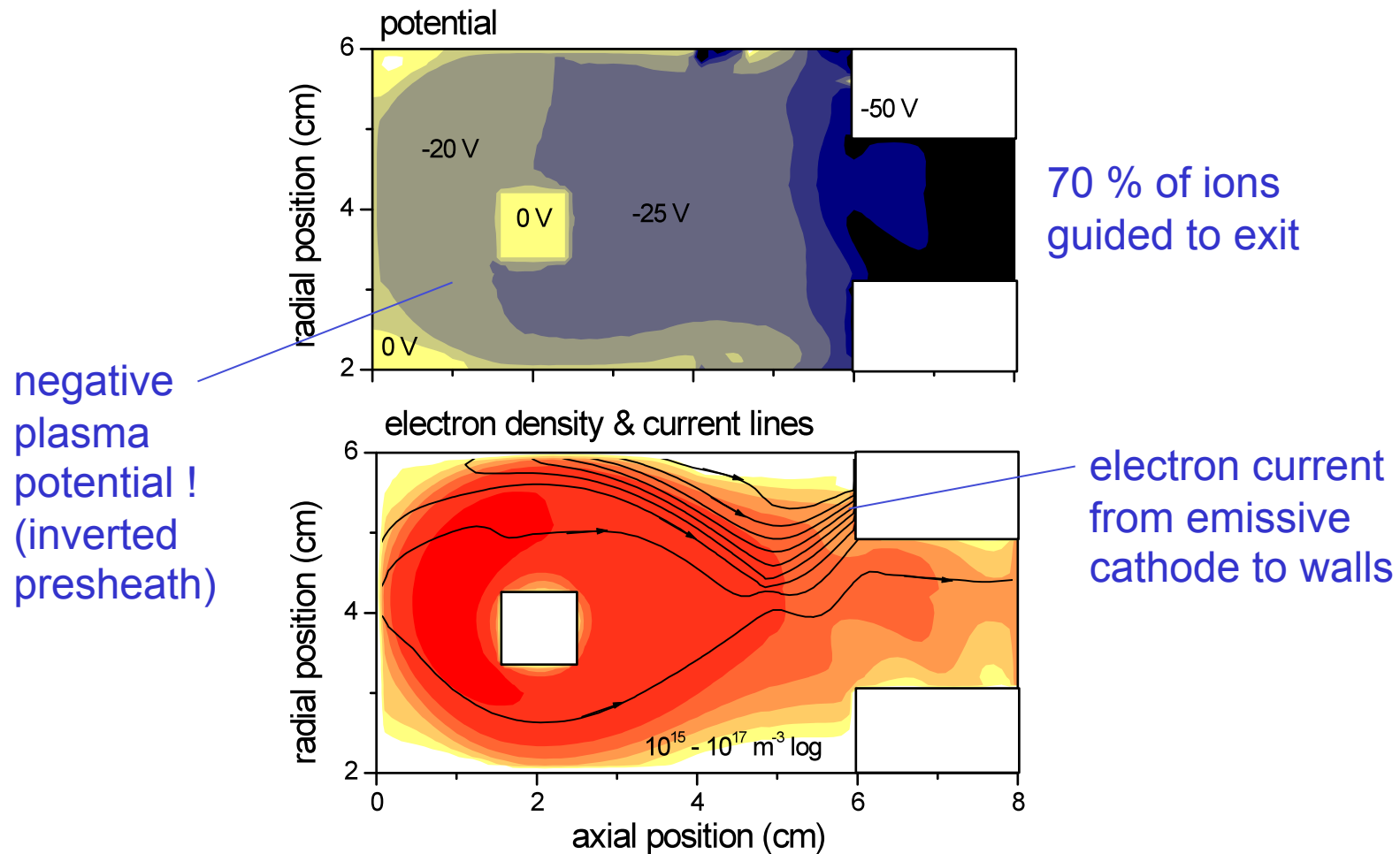
Equipotential lines  $\sim$  magnetic field lines  
Applied voltage penetrates in plasma bulk

## Example III : semi-Galathea trap



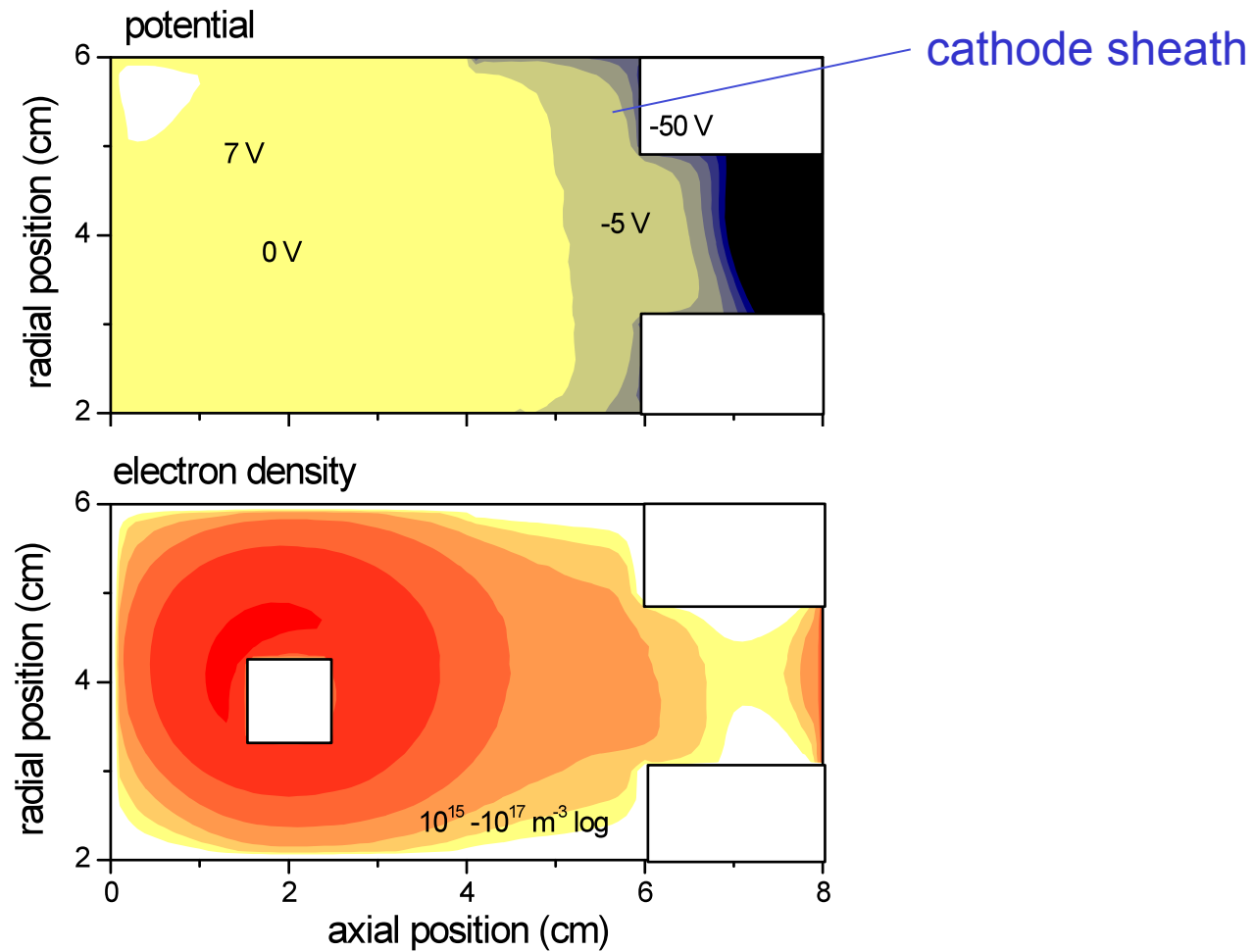
A. I. Morozov and V. V. Savel'ev, *Physics – Uspekhi* **41** (11), 1049-1089 (1998).

# Semi-Galathea trap



Potential well reduces ion wall loss and guides ions to exit

# Semi-Galathea trap without emission



Potential well disappears because of cathode sheath

# Conclusions

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- In magnetized discharges, charged particle transport and space charge fields are different
- This can be studied in 2D by hybrid models
- No predictive simulations, but insight in physical principles